

Interaction effects on quasiparticle localization in dirty superconductors

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We study how quasiparticle interactions affect their localization properties in dirty superconductors with broken time reversal symmetry – for example in a magnetic field. For $SU(2)$ spin-rotation invariant (class C) systems, the only important coupling is the spin-spin triplet interaction, which we study within a renormalization group approach. Either an additional Zeeman coupling or a complete breaking of spin rotation symmetry renders all interactions irrelevant. These two situations realize, respectively, the non-interacting unitary Anderson and the “thermal” (class D) universality class. Our results imply a stable metallic phase in 2D for class D. Experimental implications are discussed.

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The interplay between disorder and interactions in electronic systems underlies several interesting phenomena in solids. A famous, though poorly understood, example is the metal insulator transition in three-dimensional dirty solids. Although much theoretical progress has been achieved in understanding the Anderson localization transition of *non-interacting* electrons, direct contact with experiments has been problematic due to the complicating effect of interactions. Interesting field theoretic attempts have been made to incorporate interactions into the successful scaling theory of the Anderson transition [1]. However the resulting theories are complicated to analyse, and in most cases have thus far not led to a good description of real metal-insulator transitions. In two dimensional systems (2D), the situation is even worse: even the possible stability of a genuine metallic phase in the presence of disorder and interactions is a matter of considerable current debate [2]. Recent work [3,4] has made some progress in understanding the field theory of the disordered, interacting, two-dimensional electron gas.

As recently emphasized, the dynamics of quasiparticles in a superconductor provides a new, though still experimentally relevant, context to address localization issues [5–10]. All disordered superconductors fall into one of two categories according to the nature of their quasiparticle transport properties – superconducting “thermal metals” (with delocalized quasiparticles) or “thermal insulators” (with localized quasiparticles). Interesting differences arise with localization physics in a normal metal due to the lack of conservation of the quasiparticle electric charge in the superconductor.

In this paper, we take up the task of describing quasiparticle localization inside superconductors in the presence of both disorder and interactions [11]. We argue that this problem is simpler than the corresponding problem in normal metals. We explicitly identify physical situations in which the interaction effects are unimpor-

tant for the long distance physics. The corresponding experimental systems thus provide a clear opportunity to study Anderson localization transitions, unhindered by interaction effects. We also demonstrate the stability of a thermal metal phase in 2D in the presence of both interactions and disorder inside superconductors under appropriate conditions. The insights gained from studying the superconductor may be valuable in developing an understanding of the normal metal.

Within the standard mean field treatment of pairing, the dynamics of non-interacting BCS quasiparticles is governed by a quadratic Bogoliubov-deGennes (BdG) Hamiltonian, subject to static disorder in the normal and pairing potentials. The localization physics of quasiparticles has been studied previously in this approximation. A total of four universality classes, different from the three known standard classes for normal metals, have been found for the possible localization behavior of non-interacting quasiparticles inside dirty superconductors [5–7], on length scales much larger than the mean free path. These correspond to BdG Hamiltonians with or without spin-rotation and/or time reversal symmetries. Since charge is not conserved in a superconductor, the nature of a phase (metal, insulator, or Hall insulator) manifests itself not in charge transport, but rather in thermal-transport or, when spin is conserved, in concomitant spin-transport.

Here we investigate the effect of quasiparticle interactions, focusing on situations lacking time reversal invariance. One simplification offered by the superconductor (as compared to a normal metal) is that the long-range Coulomb interaction is always screened out by the condensate. Thus, the quasiparticle interaction is short-ranged. Moreover, lack of charge conservation renders the singlet density-density interaction unimportant altogether. As in normal metals, systems with broken time reversal symmetry offer the further simplification that interactions in the Cooper channel are also unimportant.

The most significant interaction then is a short-ranged interaction between the quasiparticle spin densities.

Possible experimental realizations include certain heavy fermion superconductors [12], which typically have strong spin-orbit (S.O.) scattering, and superfluid He-3 in porous media [13]; these fall into class D of [5].

Perhaps the most promising prospect is Type II superconductors in strong magnetic fields [8]. It has been suggested that a “thermal insulator-metal” transition, which can be probed by ultralow temperature heat transport measurements, may be driven in such a superconductor by simply changing the magnetic field. If the Zeeman coupling to the magnetic field and S.O. scattering can be ignored, the transition is in the universality class describing a superconductor with full $SU(2)$ spin symmetry, but without time reversal symmetry (class C of Ref. [5]). However, typical Zeeman energies are expected to be much bigger than the low temperatures necessary to clearly extract the electronic heat transport. Thus, the asymptotic critical properties will be described by a theory that includes the Zeeman coupling. In the absence of S.O. scattering this situation is formally in the same universality class as that of spinless *electrons* in a magnetic field [6]. Short-range interactions are known to be irrelevant in the latter model [14] - a physical consequence of the Pauli principle. Therefore, the field-driven thermal metal-insulator transition in a bulk 3D type II superconductor provides an excellent (and possibly unique) opportunity to experimentally study the non-interacting 3D (unitary) Anderson transition, in contrast to normal metals where the long-range Coulomb interaction changes the universality class. Despite being irrelevant, the short-range interactions in this system can still affect the low frequency, finite temperature (T) dynamics: a calculation along the lines of [15] shows that the thermal conductance κ for $T \rightarrow 0$ behaves as $\kappa/T \propto T^{\theta(d-2)}$.

Without interactions, $\theta = \nu$, the localization length exponent. With interactions, $\theta = \min(\nu, \frac{p}{2})$, where p is the “dephasing” exponent arising from the irrelevant interactions. To lowest order in the $d = 2 + \epsilon$ expansion one finds $p < 2\nu$ ($p = 1.3$ when naively extrapolated to $d = 3$.)

If, on the other hand, even uniaxial spin rotation symmetry is broken (class D of [5]) due to, for example, S.O. scattering, then for the same physical reason as above, interactions are again expected to be irrelevant. In 3D, this is thus expected to realize the thermal metal-insulator transition discussed in [7]. In 2D, this implies that the (thermal) metallic phase in non-interacting models of such superconductors [7,9,10] is stable to the inclusion of interactions. This then provides a concrete theoretical instance of a stable metallic phase in 2D, albeit for thermal transport.

To study interaction effects in classes C and D, we construct non-linear sigma models[NL σ M] which generalize those constructed by Finkelstein for normal metals [1]. Within this framework we see the irrelevance of all interactions in class D, and that of singlet and Cooper interactions in class C. For the latter case we carry out a perturbative Wilsonian renormalization group (RG) analysis: In $2 + \epsilon$ dimensions ($\epsilon > 0$) the remaining triplet interaction is found to be marginal, to 1-loop order, at the fixed point describing the (thermal) metal-insulator transition without interactions [8]. In 2D, we find that the triplet interaction strongly affects the weak localization correction to the spin- (and thermal-) conductivity in the metallic phase (at weak coupling), which changes sign for sufficiently attractive interactions.

Class C: We start with a general BCS Hamiltonian for a dirty singlet superconductor, possessing spin rotation invariance, but no time reversal symmetry, in the *absence of quasiparticle interactions*:

$$\mathcal{H} = \int d^d x \sum_{\alpha} \psi_{\alpha}^{\dagger} \left(-\frac{\nabla^2}{2m} - \mu + V(x) \right) \psi_{\alpha} + \left(\Delta(x) \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} + \text{h.c.} \right) \quad (1)$$

Here $\alpha = \uparrow, \downarrow$ labels the spin. The potential $V(x)$, as well as the real and imaginary parts of the complex gap function $\Delta(x)$, are (static) random variables with Gaussian distributions of zero mean and (without loss of generality) of same variance. The vanishing mean of the complex $\Delta(x)$ is physically appropriate in the Type II superconductor in the mixed phase with randomly located vortices, and in other physical situations. We treat the disorder average using the Schwinger-Keldysh method (described for the unitary symmetry class [16] - see also [17]). Within this formalism, the fermions acquire an additional Keldysh index $i = 1, 2$ denoting the time-ordered and anti-time-ordered branches of the path integral. At zero temperature the Keldysh functional integral action corresponding to the above Hamiltonian is (spin and Keldysh labels suppressed where possible)

$$S = \int d^d x dt \psi^{\dagger}(x, t) \sigma_z \left[i \partial_t - \frac{\nabla^2}{2m} - \mu \right] \psi(x, t) + i \eta \int d^d x \frac{d\omega}{2\pi} \text{sgn}(\omega) \psi^{\dagger}(x, \omega) \psi(x, \omega) + \int d^d x dt \left[V(x) \psi^{\dagger}(x, t) \sigma_z \psi(x, t) + \Delta(x) \psi_{\uparrow}^{\dagger}(x, t) \sigma_z \psi_{\downarrow}^{\dagger}(x, t) + \Delta^*(x) \psi_{\downarrow}^{\dagger}(x, t) \sigma_z \psi_{\uparrow}^{\dagger}(x, t) \right] \quad (2)$$

where σ_z is a Pauli matrix in Keldysh space. The second term ($\eta > 0$) specifies the time-ordering on the Keldysh contour. The symmetries are made explicit by introducing a 4-component field on each Keldysh branch,

$$\chi_i(x, t) = \begin{pmatrix} \chi_{i,a=1}(x, t) \\ \chi_{i,a=2}(x, t) \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_i(x, t) \\ i\tau_y \psi_i^\dagger(x, t) \end{pmatrix} \quad (3)$$

satisfying the reality condition $\chi_i^\dagger = (C\chi_i)^T$ with $C = \mu_y \tau_y$ (Pauli matrices $\vec{\mu}_{ab}$ act on the ‘particle-hole’ index a of χ_i , and $\vec{\tau}_{\alpha\beta}$ on the spin-index.) When expressed in terms of the 8-component fermion field $\chi(x, \omega)$, the Keldysh action above is readily seen to be invariant under independent symplectic transformations $U(\omega) \in Sp(4) \subset U(4)$ for each frequency ω (repeated indices summed),

$$\chi_{i,a,\alpha}(x, \omega) \rightarrow U_{i\alpha;j\beta}(\omega) \chi_{j,a,\beta}(x, \omega); \quad U^T \tau_y \sigma_z U = \tau_y \sigma_z$$

when $\eta = 0$. A finite η breaks the symmetry down to a $U(2)$ subgroup. By standard arguments the low energy degrees of freedom are described by a diffusion mode matrix field $Q(x) = Q_{\omega_1, \omega_2}^{ij; \alpha, \beta}(x)$ which decouples fermion bilinears $\sigma_z^{ij}(\chi_{i\alpha}^\dagger(x, \omega_1) \chi_{j\alpha\beta}(x, \omega_2))$, appearing after the disorder average. It carries spin (α, β) , Keldysh (i, j) , and frequency indices. Q takes values on the saddle point manifold specified in Eq. (7) below, and its dynamics is governed by the Keldysh NL σ M action Eq. (5). This formulation of the non-interacting disordered theory is entirely equivalent to those obtained using the replica trick or supersymmetry.

Effect of interactions: In the absence of disorder, the interactions at the Fermi surface can be, most generally [18], of density-density (singlet), spin-spin (triplet) and (singlet and triplet) Cooper types. Introducing three Hubbard-Stratonovich (H.S.) fields to decouple these four-fermion interactions, and condensing the H.S. field which decouples the Cooper interactions (gap function) in the spin-singlet channel, gives the BCS mean field theory. In the presence of disorder, but no interactions, this has the form of Eq.(1). Quasiparticle interactions arise from the (dynamical) fluctuations of the three H.S. fields. In particular, residual interactions in the Cooper channel arise from the fluctuations of the amplitude and the phase of the gap function. The gapless phase fluctuations can be seen to decouple from the BdG quasiparticles at asymptotically low energies, and their only effect is to render a long-range Coulomb interaction short-ranged [19]. This confirms the expected screening by the condensate, mentioned above. Averaging over disorder in the presence of the three H.S. fields yields, along the lines of [16], a theory of diffusion modes Q interacting with the latter. In the absence of fluctuations of the gap function amplitude, the remaining two H.S. fields can be integrated out straightforwardly. The singlet H.S. field is seen to decouple, and integration over the triplet H.S. field leads to an interaction term of the familiar

Finkelstein form. In particular, the resulting Keldysh-Finkelstein action reads (after simple rescalings and unitary transformations):

$$Z = \int [DQ] e^{-S[Q]}; \quad S[Q] = S_D[Q] + S_{\text{int}}[Q] \quad (4)$$

$$S_D[Q] = \int \frac{d^d x}{4} \left[\frac{1}{8\pi g} \text{Tr}(\nabla Q)^2 + 4z \text{Tr}(i\omega \sigma_z - \eta \text{sign}(\omega)) Q \right] \quad (5)$$

$$S_{\text{int}}[Q] = i\pi U_t z^2 \sum_i \sigma_z^{ii} \int d^d x \int dt Q_{t,t}^{ii, \alpha\beta} Q_{t,t}^{ii, \beta\alpha} \quad (6)$$

Here Tr denotes the trace over all indices of Q , including the frequency index ω , and $Q_{t_1, t_2}^{ii, \alpha\beta}$ is the corresponding (double-) time Fourier transform. The saddle point manifold of massless modes of the NL σ M is described in frequency space by

$$Q^2 = 1, \quad Q^\dagger = Q, \quad \text{Tr}(Q) = 0; \quad \tau_y \Sigma_x Q \tau_y \Sigma_x = -Q^T \quad (7)$$

where Σ_x exchanges positive and negative frequencies. Note that $Q_{t,t}^{ii, \alpha\beta}$ transforms as a spin-triplet. Consequently, Eq.(6) is the only $SU(2)$ spin-rotation invariant Finkelstein interaction term that can be written. This was expected: due to the lack of charge conservation there is no massless charge diffusion mode, and due to the lack of time-reversal symmetry, there is no massless Cooperon mode on the saddle point manifold. Hence Eq.(6) represents the only non-vanishing interaction, which is that between the spin diffusion modes. Here $1/g$ is proportional to the (spin) conductivity, and $U_t (< 0)$ is the (repulsive) triplet interaction strength.

We perform a perturbative (Wilsonian) R.G. analysis, parametrizing the saddle point manifold by independent matrix fields V , similar to [16]. The wavefunction renormalization is found by evaluating $\langle Q \rangle$, using the same logic as in the standard $O(N)$ NL σ M [20]. The 1-loop R.G. equations, valid to lowest order in g (and z), but to all orders in $U_t z$, are found in $d = 2 + \epsilon$:

$$\frac{dg}{dl} = -\frac{\epsilon}{2}g + \left[7 - 6 \left(1 - \frac{1}{2U_t z} \right) \log(1 - 2U_t z) \right] g^2 + O(g^3)$$

$$\frac{d(U_t z)}{dl} = O(g^2), \quad \frac{dz}{dl} = \left[\frac{\epsilon}{2} - (1 + 6U_t z)g \right] z + O(g^2)$$

(l is twice the log of the length scale). These are extracted from the renormalization of the terms quadratic in V in the action. We also confirmed that the same R.G. equations are obtained from the renormalization of the terms of next higher order in V . This is a necessary condition for the renormalizability of the theory.

Note that in contrast to other universality classes, $U_t z$, a measure of the interaction strength, does not renormalize to 1-loop order in class C.

Without interactions ($U_t = 0$), we recover the non-interacting fixed point at $g = \frac{\kappa}{2} + \dots$, which is the thermal (or spin) metal-insulator transition considered in [8]. We see from the second R.G. equation that the interaction $U_t z$ is marginal at this fixed point, at least to 1-loop order. Hence, showing that interactions are relevant or irrelevant would require working to higher-loop order.

Next we specialize to 2D, focussing on the metallic phase where $g \ll 1$. Since $U_t z$ does not renormalize to lowest order in g , it will be constant over a wide range of length scales. Hence, in this range, the thermal- (κ), and spin- (σ^s) conductivity is given by the 1-loop result ($\frac{3\hbar^2}{4\pi^2 k_B^2} \frac{\kappa}{T} = \sigma^s$):

$$\sigma^s = \sigma_0^s - \frac{1}{4\pi^2} \left[7 - 6 \left(1 - \frac{1}{2U_t z} \right) \log(1 - 2U_t z) \right] \log \left(\frac{L}{\ell_e} \right)$$

Here σ_0^s is the (bare) spin conductivity at the scale of the elastic mean-free path ℓ_e , and σ^s its value at length scale L . This expression is perturbative in $g = (\frac{2}{\pi})1/\sigma^s$, but $U_t z$ need not be small. (Note that the quantity in square brackets goes to 1 as interactions are removed.) Thus, a repulsive (attractive) triplet interaction is seen to decrease (enhance) the weak localization correction to the (spin-, or thermal-) conductivity, as compared to the non-interacting case. For sufficiently strongly repulsive interactions, $2U_t z < -0.37\dots$, the g^2 -coefficient in dg/dl changes sign, implying that g flows back to weak coupling, at least until, potentially, higher-loop effects set in which may reverse the flow.

Class D: Turning now to situations lacking spin-rotation as well as time reversal symmetry, we can see, without extensive calculations, that all Finkelstein-type interaction terms are absent. In this case, the diffusion mode matrix field $Q_{t_1, t_2}^{ij}(x)$ decoupling fermion bilinears $\sigma_z^{ij}(\chi_{ia\alpha}^\dagger(x, t_1)\chi_{ja\alpha}(x, t_2))$ carries only Keldysh (i, j) and frequency, but no spin indices. Repeating the steps above Eq.(4), both the singlet and triplet H.S. fields are seen to decouple. This was expected since now, due to lack of spin-rotational symmetry, also the spin diffusion mode is absent from the saddle point manifold of massless modes. Indeed, the only possible Finkelstein interaction term as in Eq.(6) (but without spin indices, α, β) vanishes due to the antisymmetry of $Q_{t_1, t_2}^{ii}(x)$, which follows from the reality condition below Eq.(3).

Consequently, we conclude that the 3D thermal metal-insulator transition in class D [7] is expected to be unmodified by quasiparticle interactions, as mentioned above. In 2D, in the absence of interactions, a (thermal) metallic phase, stable to quantum interference, can exist inside generic superconductors lacking spin rotation and time reversal invariances [7,9]. (For spin-polarized versions, see Ref. [10].) The absence of any type of marginal Finkelstein interaction terms in class D, found above, thus implies that this 2D metallic phase remains stable even upon inclusion of any type of interactions.

We end our discussion with a general caveat. The question as to whether or not additional potentially ‘dangerous’, i.e. relevant or marginal, long-range (in time) versions of the interaction terms are eventually generated upon the R.G., hinges upon a proper understanding of the renormalizability of the whole class of Finkelstein-type theories in general, which is currently lacking. However, it is not expected that initially short-range interactions would generate such long-range ones, upon integration over short-distance, short-time fluctuations in an R.G. transformation. These issues will be addressed in more detail in Ref. [19].

After our work was completed, and some of our results were reported in [11], we learned about the work by Fabrizio *et al.* [21], whose results for the correction to the 2D conductivity and density of states for class C are in agreement with ours.

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